[Time: 3:00 Hrs.] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B:
1. **All questions are compulsory.**
2. **Figures to the right indicate full marks.**
3. **Scientific calculator can be used.**

Q.1 a) Let V be a finite-dimensional vector space over the field F. Define for each $\alpha \in V, L_{\alpha(f)} = f(\alpha)$ for $f \in V^*$. Prove that the mapping $\alpha \to L_\alpha$ is and isomorphism of V onto V^{**}. **10**

b) Attempt **any Two** of the following: **10**

i) Show that the vectors $x_1 = (1,1,0)$ and $x_2 = (1, i, 1 + i)$ are in the subspace W of $\mathbb{C}^3$ spanned by $(1,0, i)$ and $(1 + i, 1, -1)$, and that x_1 and x_2 form a basis of W. **5**

ii) Let V be a vector space over the field F. Show that the intersection of any collection of subspaces of V is a subspace of V. **5**

iii) Show that solution set of Homogeneous system of linear equations is closed under addition and scalar multiplications. **5**

Q.2 a) Let A be a $n \times n$ real matrix with all diagonal entries positive, all non-diagonal entries negative and row sums are all positive. Prove that $det(A) \neq 0$. **10**

b) Attempt **any Two** of the following: **10**

i) Let $c_1, c_2, \ldots, c_n$ be column vectors of $\mathbb{R}^n$. Then prove that they are linearly dependent if and only if $\det(c_1, c_2, \ldots, c_n) = 0$. **5**

ii) Define eigenvalue and eigenvector of the matrix and hence find it of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. **5**

iii) Prove that the characteristic and minimal polynomial of linear transformation T over n −dimentional vector space V have same roots except for the multiplicities. **5**

Q.3 a) Explain Jordan Canonical Form for 3×3 matrices with suitable example. Find the Jordan Canonical Form for the matrix $\begin{bmatrix} 9 & 4 & 5 \\ -4 & 0 & -3 \\ -6 & -4 & -2 \end{bmatrix}$. **10**

b) Attempt **any Two** of the following: **10**

i) Find all matrices that commute with the following square matrix $A = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$. **5**

ii) Find the basis in which matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ 2 & -3 & 2 \end{pmatrix}$ has triangular form and find that triangular form. **5**

iii) Let T be a linear transformation on finite dimensional vector space V such that minimal polynomial for T is a product of linear factors $p = (x-\alpha_1)^{r_1}(x-\alpha_2)^{r_2} \dots \dots . . (x-\alpha_k)^{r_k}$ where $\alpha_i \in F$. Let W be a proper subspace of V which is invariant under T. Then show that there exists a vector $v \in V$ such that v is not in W and $(T - \alpha I)v$ is in W, for some characteristic value α. **5**

Q.4 **a)** Define a bilinear form. Prove that a bilinear form is reflexive if and only if it is either symmetric or alternating. **10**

b) Attempt **any Two** of the following: **10**

i) Let f be any bilinear form on a finite n–dimensional vector space V. Let W be the sub space of all β such that $f(\alpha, \beta) = 0$ for every α, show that $rank\ f = dim\ V - \ dim\ W$. **5**

ii) A bilinear form on $\mathbb{R}^2$ be defined as **5**

$$< (x_1, x_2), (y_1, y_2) > = 2x_1y_1 - 3x_1y_2 + x_2y_2$$

Find the matrix A of this bilinear form with basis $\{(1,0\}, (1,1)\}$.

iii) For the matrix $A = \begin{pmatrix} 1 & -3 & 2 \\ -3 & 7 & -5 \\ 2 & -5 & 8 \end{pmatrix}$ find non-singular matrix P such that $P^t AP$ is diagonal and also find its signature. **5**
